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# Measurement of the Thickness of Metal Plate by Ultrasonic Harmonic Method. III On the Resonance of Lower Harmonic Order

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## Synopsis

When the harmonic order of the thickness resonance is extremely low as fundamental or second, it is distinctly seen that the apparent velocity of sound computed from the ordinary relation for the resonance tends to increase somewhat. In this paper, an interpretation applicable to the resonance of such a lower order is given by introducing the phase shift of sound wave reflected internally within the metal plate, referring to the results experimentally obtained.

## I. Introduction

In the measurement of the thickness of metal plates by ultrasonic harmonic method, the relation between the thickness of metal plate  $t$  and the thickness resonance frequency  $f_n$  is generally given as follows:

$$t = n \frac{c_0}{2f_n} \quad (n = 1, 2, 3, \dots) \quad (1),$$

and

$$t = \frac{c_0}{2(f_{n+1} - f_n)} \quad (2),$$

where  $c_0$  is the velocity of the longitudinal waves which is transmitted through the metal plate to be tested, and is expressed by the following well-known relationship:

$$c_0 = \sqrt{\frac{E}{\rho} \frac{(1 - \sigma)}{(1 + \sigma)(1 - 2\sigma)}} \quad (3),$$

where  $E$  is Young's modulus,  $\rho$  density and  $\sigma$  Poisson's ratio. And this can be evaluated by measuring the resonant frequencies of vibration in the thickness direction of the plate of which the thickness is previously known. According to the experiments by the writers, the velocity for steel was determined to be 5870 m/sec, and for glass to be 5770 m/sec, as the most probable value.

As previously reported,<sup>(1)</sup> however, it should be noted here, that when the harmonic order of the thickness resonance is extremely low as fundamental or second, observed resonance frequency  $f'_n$  tends to increase somewhat as compared with the value  $f_n$  computed by taking  $c_0 = 5870$  m/sec in eq. (1). In such a case, therefore, the proper value of the thickness cannot be determined.

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(1) S. Tanaka, Sci. Rep. RITU, A 3 (1951), 201.

Since the variable frequency range of the driving oscillator has previously been fixed, when the thickness of metal plate is extremely thin, harmonic order of the resonance should necessarily be low. It is not desirable, however, to employ higher driving frequency in order to keep away from the resonance of lower order, because sound wave can hardly be transmitted into the metal plate, and also even for small indentation of the back surface of the plate, it may be impossible to obtain the resonance indication.

In this paper, the writers desire to propose an interpretation applicable to the resonance of such a lower order, referring to the results experimentally obtained.

## II. Expression for phase shift

The reason why observed resonance frequencies in lower harmonic orders increase as compared with that of higher orders, will be considered from various standpoints. As one of the writers previously pointed out,<sup>(2)</sup> however, the standing wave should be set up within the metal plate only, because a quartz plate is loosely coupled with the metal plate due to the stiffness of an oil film being wetted between these two plates.

In 1950, H. J. McSkimin<sup>(3)</sup> suggested that the phase of sound wave reflected internally within the metal plate should be shifted at the reflecting boundary to that of incident wave, and he measured directly this phase angle  $\varphi$  by means of a pulse technique.

Introducing this factor here, the expression for the phase given by him will be followed also in this case. Then it will be

$$[2t(2\pi f'_n)/c_0] + 2\varphi = 2\pi n \quad (4),$$

or

$$c_0 = \frac{2tf'_n}{n - \left(\frac{\varphi}{\pi}\right)} \quad (5).$$

From this relation, it is understood that the apparent velocity of sound is to increase when  $\varphi/\pi$  cannot be neglected as compared with  $n$ .

## III. Experimental results

In Table 1, the resonance frequencies  $f'_n$  observed for steel plates of which thickness was comparatively thin, and the values of  $\varphi/\pi$  computed from eq. (4) by putting  $c_0 = 5870$  m/sec are shown. And also the apparent velocities  $c'$  are computed from relationship (1) with observed frequencies  $f'_n$ . From these results, the value of  $\varphi/\pi$  for steel will be determined to be about  $-0.1$  radian.

Fig. 1 shows the relation between apparent velocity  $c'$  and harmonic order of resonance  $n$ , where points of crosses, triangles and circles show observed values for the plate of 2 mm, 3 mm and 4 mm in thickness, respectively. The solid line is what has been computed from the following equation by taking  $\varphi/\pi$  as  $-0.1$  radian.

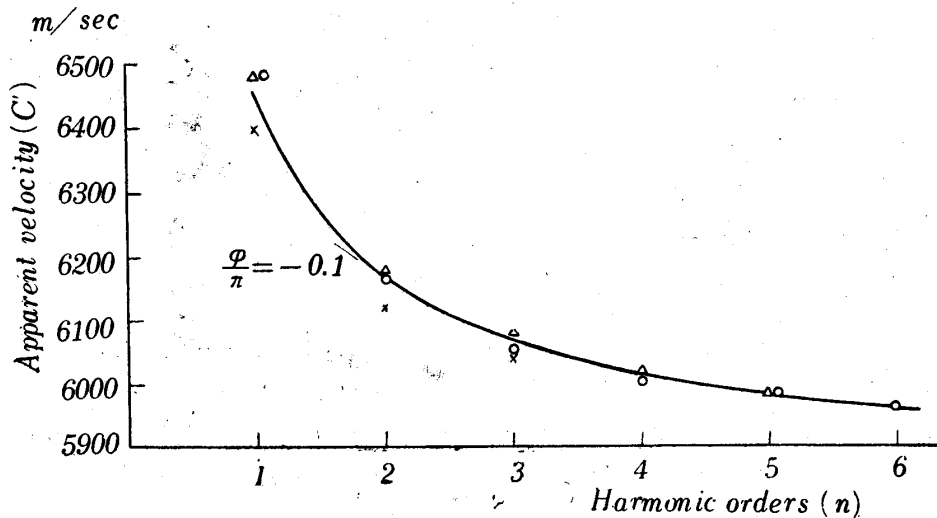
(2) S. Tanaka, Sci. Rep. RITU, A 2 (1950), 917.

(3) H. J. McSkimin, J. Acous. Soc. Am. 22 (1950), 413.

Table 1. Observed resonance frequencies, values of  $\varphi/\pi$  and apparent velocities for steel plate.

$$c_0 = 5870 \text{ m/sec}$$

Thickness of steel plate	2 mm			3 mm			4 mm		
Harmonic order	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)
$f_1$ : Funda.	1.60	-0.090	6400	1.08	-0.104	6480	0.81	-0.104	6480
$f_2$ : Second	3.06	-0.085	6120	2.06	-0.106	6180	1.54	-0.099	6160
$f_3$ : Third	4.53	-0.087	6040	3.04	-0.107	6080	2.27	-0.094	6050
$f_4$ : Fourth				4.01	-0.099	6020	3.00	-0.089	6000
$f_5$ : Fifth				4.98	-0.090	5980	3.74	-0.097	5980
$f_6$ : Sixth							4.47	-0.092	5960

Fig. 1. Relation between apparent velocity  $c'$  and harmonic order  $n$  for steel.Table 2. Observed resonance frequencies, values of  $\varphi/\pi$  and apparent velocities for glass plate.

$$c_0 = 5770 \text{ m/sec}$$

Thickness of glass plate	2 mm			3 mm			4 mm		
Harmonic order	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)	Obs. res. freq. $f'$ (MC)	$\varphi/\pi$ (radian)	App. velocity $c'$ (m/sec)
$f_1$ : Funda.	1.50	-0.040	6000	1.00	-0.040	6000	0.755	-0.047	6040
$f_2$ : Second	2.96	-0.052	5920	1.97	-0.048	5910	1.48	-0.052	5920
$f_3$ : Third	4.36	-0.021	5810	2.92	-0.036	5840	2.18	-0.021	5810
$f_4$ : Fourth	5.81	-0.028	5810	3.88	-0.035	5820	2.90	-0.021	5800
: Sixth				4.84	-0.035	5810	3.63	-0.033	5808

$$c' = \frac{n - \left( \frac{\varphi}{\pi} \right)}{n} c_0 \quad \dots\dots\dots (5).$$

Next, for the purpose of comparison, the similar values for glass were measured. Specimens were prepared from a plate produced by the Asahi Glass Co., and pieces 25 mm in diameter were carefully ground and polished before testing. Table 2 shows the observed resonance frequencies  $f'_n$ , the values of  $\varphi/\pi$ , and the apparent velocities  $c'$ ,

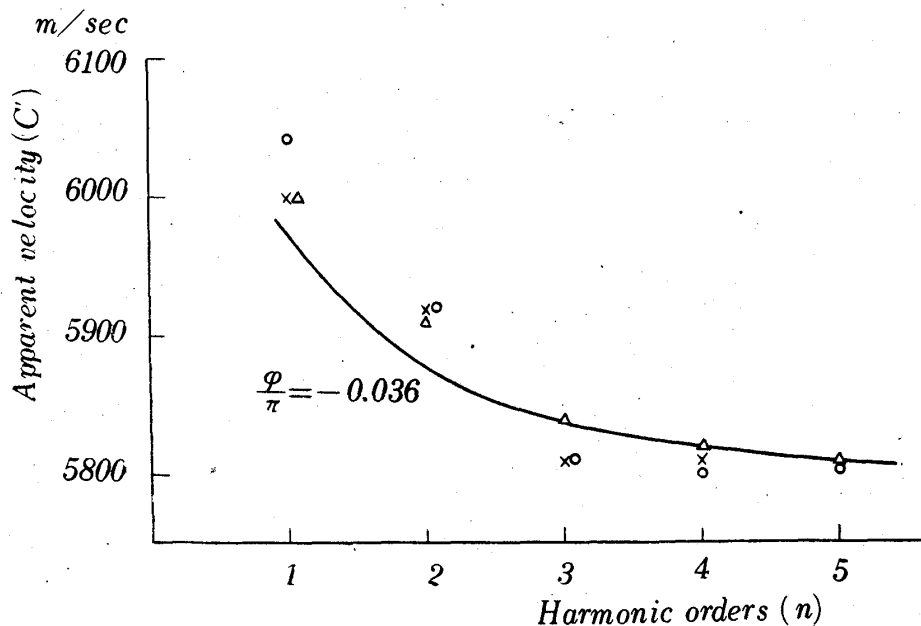


Fig. 2. Relation between apparent velocity  $c'$  and harmonic order  $n$  for glass.

respectively, obtained for glass in the same way as in the case of steel. In this case, the value of  $\varphi/\pi$  was determined to be about  $-0.036$  radian, taking  $c_0$  as 5770 m/sec.

The solid line in Fig. 2 shows the relation computed from eq. (4) by putting  $\varphi/\pi = -0.036$  radian. Resonance frequencies observed with plates of 2 mm, 3 mm and 4 mm in thickness did not fairly agree with the computed line as in the case of steel.

From these results, it may be seen that the increase of the apparent velocity of sound wave at the resonance of extremely lower order can be explained by introducing a conception with the phase shift at the reflecting boundary.

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